

# The Challenge of Unifying Semantic and Syntactic Inference Restrictions

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## Abstract

While syntactic inference restrictions don't play an important role for SAT, they are an essential reasoning technique for more expressive logics, such as first-order logic, or fragments thereof. In particular, they can result in short proofs or model representations. On the other hand, semantically guided inference systems enjoy important properties, such as the generation of solely non-redundant clauses. I discuss to what extent the two paradigms may be unifiable.

## 1 Introduction

In [21] I discussed the differences between simple-syntactic portfolio solvers and sophisticated-semantic portfolio solvers. In this paper I present the challenges in combining a solver based on syntactic inference restrictions with a solver based on the semantic guidance of inferences. More concretely, I investigate the differences between CDCL-style solvers building a partial model assumption and solvers based on ordering and selection restrictions on inferences.

## 2 NP-Complete Problems

The prime calculus for SAT is CDCL (Conflict Driven Clause Learning) [18, 13, 4, 5]. The calculus can be viewed as a resolution variant where resolution inferences are selected via partial model assumptions. The CDCL calculus operated on a five tuple  $(M, N, U, k, C)$  where  $M$  is a sequence of literals representing the current model assumption, called the *trail*,  $N$  the clause set under consideration,  $U$  a set of learned clauses, i.e., clauses derived via resolution,  $k$  the number guessed/decided literals in  $M$ , called the *decision level*, and  $C$  a clause that is  $\top$  if no conflict has occurred yet, some non-empty clause representing a found conflict, or  $\perp$  in case of an overall contradiction. Consider the clause set

$$N = \{P \vee Q \vee R, \neg R \vee S, \neg S \vee P \vee Q, \dots\}$$

where a CDCL run starting from the empty trail  $\epsilon$  may result in

$$\begin{aligned} &(\epsilon; N; \emptyset; 0; \top) \\ &\Rightarrow_{\text{CDCL}}^* ([\neg P^1 \neg Q^2 R^{P \vee Q \vee R} S^{\neg R \vee S}]; N; \emptyset; 2; \neg S \vee P \vee Q) \end{aligned}$$

where the literals  $\neg P$ ,  $\neg Q$  were decided (guessed) at decision levels 1, 2, respectively;  $R$  is propagated via the clause  $P \vee Q \vee R$ ;  $S$  is propagated via  $\neg R \vee S$  and finally in the partial assignment  $[\neg P^1 \neg Q^2 R^{P \vee Q \vee R} S^{\neg R \vee S}]$  the clause  $\neg S \vee P \vee Q$  is false. The conflict is solved by resolving the false clause with clauses propagating on the trail. First with  $\neg R \vee S$  resulting in

$\neg R \vee P \vee Q$  and then with  $P \vee Q \vee R$  resulting in  $P \vee Q$ . Now this clause is learned yielding the new CDCL state

$$\Rightarrow_{\text{CDCL}}^* ([\neg P^1 Q^{P \vee Q}]; N; \{P \vee Q\}; 1; \top).$$

Most importantly, the new clause  $P \vee Q$  is *non-redundant*, i.e., it is not implied by smaller clauses from  $N$ , where the ordering is a lifting of the literal ordering generated by the trail [1, 10]. Propositional non-redundancy is itself an NP-complete property. Hence, a polynomial time procedure generates a clause that enjoys an NP-complete property. This has several immediate consequence: (i) termination (ii) the approach of forgetting clauses works. Note that since only non-redundant clauses are learned by CDCL, a forgotten clause that has become redundant will not be generated a second time. Hence, learning plus forgetting can be seen as an efficient way to getting rid of redundant clauses.

Please recall, the classical first-order notion of redundancy means “not needed to find a proof or model”. Therefore, a redundant clause can be deleted. In the superposition or ordered resolution context this abstract definition is instantiated with “not implied by smaller clauses”. However, in the context of SAT, redundancy is often defined as a synonym for “satisfiability preserving”. So in the context of many SAT related papers redundant clauses must not be removed, in general, or completeness is lost. These two notions of redundancy must not be confused and I stick here to the classical first-order notion.

The prerequisites for learning non-redundant clauses are twofold:

1. Propagation needs to be exhaustive.
2. Conflict detection needs to be eager.

The number of literals of a SAT problem is typically small compared to the respective clause set size. Therefore, exhaustive propagation in SAT is not an issue, as well as the detection of conflicts, i.e., false clauses. Surprisingly, what holds for SAT, does not hold for NP-complete problems, in general. It seems that the language of propositional logic is a nice compromise between expressivity and the efficiency of propagation.

Consider the NP-complete problem of testing satisfiability of a system of linear arithmetic inequations over the integers [11, 14]. The language of linear integer arithmetic is more succinct than propositional logic, i.e., the encoding of an integer variable  $x$  requires linearly many propositional variables, because the a priori bounds for solvability are simply exponential. Now consider a CDCL style procedure testing solvability of a LIA (Linear Integer Arithmetic) system of inequations [8]. Consider the example system

$$N = \{1 - x - y \leq 0, x - y \leq 0, \dots\}$$

and a CDCL-style run via CATSAT+ [8], where the partial model assumption is represented by simple bounds, i.e., inequations of the form  $x \# c$ ,  $c \in \mathbb{Z}$ ,  $\# \in \{<, >, \leq, \geq\}$ .

$$\begin{aligned} &(\epsilon; N; \emptyset; 0; \top) \\ &\Rightarrow_{\text{CATSAT}^+}^* ([x \geq 5^1 y \geq 6^{1-x-y \leq 0} x \geq 6^{x-y \leq 0} \dots]; N; \emptyset; 1; \top) \end{aligned}$$

Obviously, the propagation does not terminate, except for a priori simply exponential bounds: if  $m$  is the number of inequations in  $N$ ,  $n$  the number of different variables in  $N$  and  $a$  the maximal coefficient in  $N$ , then the problem has a solution iff  $-n \cdot (m \cdot a)^{2m+1} \leq x \leq n \cdot (m \cdot a)^{2m+1}$  for every variable  $x$  in  $N$  [14]. Hence, there is currently no CDCL-style procedure known that does exhaustive propagation. Therefore, the learned clauses [8] are not

non-redundant. The current state-of-the-art procedures don't use a CDCL-style approach but apply a relaxation of LIA to LRA (linear rational arithmetic) and use LRA solutions for a branch-and-bound approach. This general idea is complemented via simplifications and fast, sufficient tests for the existence of a solution [6, 12, 9, 7].

### 3 NEXPTIME-Complete Problems

The same phenomenon that occurs at LIA, see Section 2, also shows up if the complexity is “slightly” increased from SAT to the satisfiability of the Bernays-Schoenfinkel (BS) fragment of first-order logic [3], which is NEXPTIME-Complete [17]. For example, consider the following clause set [15]

$$N = \left\{ \begin{array}{l} 1 : P(0, 0, 0, 0), \\ 2 : \neg P(x_1, x_2, x_3, 0) \vee P(x_1, x_2, x_3, 1), \\ 3 : \neg P(x_1, x_2, 0, 1) \vee P(x_1, x_2, 1, 0), \\ 4 : \neg P(x_1, 0, 1, 1) \vee P(x_1, 1, 0, 0), \\ 5 : \neg P(0, 1, 1, 1) \vee P(1, 0, 0, 0), \\ 6 : \neg P(1, 1, 1, 1) \end{array} \right\}$$

where a CDCL-style run, using ground literals for the partial model assumptions [10], will propagate all values of the 4-bit counter represented by  $P$ :

$$\begin{aligned} & (\epsilon; N; \emptyset; 0; \top) \\ & \Rightarrow_{\text{SCL}}^* ([P(0, 0, 0, 0)^1 P(0, 0, 0, 1)^2 P(0, 0, 1, 0)^3 \dots P(1, 1, 1, 1)^2], N, \emptyset, 0, \neg P(1, 1, 1, 1)) \end{aligned}$$

before detecting the conflict, where for the propagation justifications I used the clause numbers. Thus, there are exponentially many propagations, in general. Still, the clause set can be refuted in linearly many steps by starting with resolution steps between the clauses 2 – 4:

$$\begin{array}{ll} 2.2 \text{ Res } 3.1 & 7 : \neg P(x_1, x_2, 0, 0) \vee P(x_1, x_2, 1, 0) \\ 7.2 \text{ Res } 2.1 & 8 : \neg P(x_1, x_2, 0, 0) \vee P(x_1, x_2, 1, 1) \\ 8.2 \text{ Res } 4.1 & 9 : \neg P(x_1, 0, 0, 0) \vee P(x_1, 1, 0, 0) \\ 9.2 \text{ Res } 8.1 & 10 : \neg P(x_1, 0, 0, 0) \vee P(x_1, 1, 1, 1) \\ 10.2 \text{ Res } 5.1 & 11 : \neg P(0, 0, 0, 0) \vee P(1, 0, 0, 0) \\ 11.2 \text{ Res } 10.1 & 12 : \neg P(0, 0, 0, 0) \vee P(1, 1, 1, 1) \\ 12.1 \text{ Res } 6.1 & 13 : \perp. \end{array}$$

which corresponds to a standard ordering restriction on the  $P$  atoms, e.g., via a KBO, plus a respective selection strategy [2]. For example, for the first step the second literal out of clause 2 is maximal with respect to a KBO instance where  $1 >_{\text{KBO}} 0$  and the first literal of clause 3 is selected.

In addition, for “practically relevant” instances of the BS fragment the situation is the same. We have shown [20] that the YAGO [19] fragment of BS can be effectively saturated by a variant of ordered resolution with chaining, whereas all model-guided approaches fail. Again for the reason that propagation with respect to millions of constants cannot be efficiently done, yet.

## 4 Unification

The two frameworks, syntactic inference restrictions and model-guided inferences cannot be easily combined. Obviously, since ordered resolution may generate redundant clauses, but model-guided inferences with exhaustive propagation and eager conflict detection do not, model-guided inferences cannot simulate resolution inferences. The resolution inferences in Section 3 refuting the counter encoding are not redundant. Still, they cannot be simulated by a CDCL-style calculus [10] because it will immediately run into the exponentially many propagation steps and find a refutation of exponential size this way. It seems to be non-trivial and non-obvious how the two paradigms can be unified. If eager propagation is dropped, then model-guided inferences can simulate resolution [16, 10], however, in this case non-redundancy of learned clauses is lost, in general.

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